1 Discussion: Brock, William A. and Steven N. Durlauf (2001).

1.1 Goals of the Paper

1. Generalized logistic model of discrete choice with social interactions
2. Study aggregate behavioral outcomes when individuals' utility functions exhibit social interaction effects
3. Analytical framework is a random field model: Law of large numbers for the distribution of choices in the population
4. Equilibrium and its properties: multiple equilibria, local stability
5. Efficiency: Comparison of a noncooperative economy versus social planner
6. Econometrics: Likelihood function, conditions under which the model is identified

1.2 Introduction

1.2.1 Social Interactions and Applications

> By social interactions, we refer to the idea that the utility or payoff an individual receives from a given action depends directly on the choices of others in that individual's reference group, as opposed to the sort of dependence which occurs through the intermediation of markets. This type of spillover is an example of a classical nonpecuniary externality. (p. 235)

Notice that this type of interaction is different from the models of informational externalities. Here, the choices of others influence directly one's utility while in Banerjee (1992) one only infers from the actions of others whether it is desirable. Some examples of applications in economics of this type of reasoning

1. Case and Katz (1991): Social ills in neighborhoods
2. Crane (1991): school dropout and teenage childbearing depends on occupational composition of community
3. Havemann and Wolfe (1994): school dropout
4. Glaeser, Sacerdote and Scheinkman (1996): community crime rates
5. Brock (1993): asset market volatility

No new idea in sociology: "Role models"
General results:

1. Tendency towards common behavior or few polarized types of behavior
2. strategic complementarities: multiple equilibria
3. Social multiplier: small changes in the private utility of an action can potentially have big effects.

1.3 The Model

1.3.1 Utility Function

There are I agents who simultaneously face a binary choice problem (ω_i) from a choice set $\{-1, 1\}$. Let $\omega = (\omega_1, ..., \omega_I)$ denote the vector of choices and $\omega_{-i} = (\omega_1, ..., \omega_{i-1}, \omega_{i+1,...}\omega_I)$ denote the choices of all agents other than i. Let $m^e_{i,j}$ denote the subjective expected value of agent i of agent j's choice, $\overline{m}^e_i = \frac{1}{I-1} \sum\limits_{j \neq i} m^e_{i,j}$ denotes the average over all agents other than i..

The utility function is the same for all agents and given by

$$V(\omega_i) = u(\omega_i) + S(\omega_i, \mu^e_i(\omega_{-i})) + \epsilon(\omega_i)$$

where $u(\omega_i)$ is the (deterministic) private utillity of a choice to the agent. $S(\omega_i, \mu^e_i(\omega_{-i}))$ is the social utility. It depends on one's own action and the conditional probability measure that agent i places on the choices of others. As usual in discrete choice models $\epsilon(\omega_i)$ denotes an i.i.d random utility term. The agent knows the value of this term but not the econometrician.

1.3.2 Parametric Assumptions

The first assumptions are about the nature of the social interactions. It is assumed that agents care about the average choices of the other agents in the respective neighborhood where the identity of the other agents is not important. That is the interactions are assumed to be **totalistic**, all interactions are treated symmetrically. There are no 'best buddies' who might be especially influential on your decision. The interactions are also to be assumed to be **strategic complementarities** and **constant**. These assumption give rise to the following

$$\frac{\partial^2 S(\omega_i, \overline{m}^e_i)}{\partial \omega_i \partial \overline{m}^e_i} = J > 0$$

They propose two different functional forms that satisfy these requirement:

1. Proportional Spillovers Case: $S(\omega_i, \overline{m}^e_i) = J\omega_i \overline{m}^e_i$
2. Conformity Effects: $S(\omega_i, \overline{m}^e_i) = -\frac{J}{2}(\omega_i - \overline{m}^e_i)^2$

Finally, the random error term is independent and extreme-value distributed. Therefore,

$$\Pr(\epsilon(-1) - \epsilon(1) \leq x) \frac{1}{1+\exp(-x)}$$

1.3.3 Equilibrium for the Proportianal Spillovers Case

The agents act simultaneously and noncooperatively. There is no communication or coordination.

> Operationally, this means that each agent makes his choice given an expectation of the mean choice level which is independent of the realizations of $\epsilon(\omega_i)\, \forall i$. (p.239)

Given the distributional assumption the probability of an action is given by:
$\Pr(\omega_i) = \frac{\exp(\beta(u(\omega_i)+J\omega_i\overline{m}_i^e))}{\sum\limits_{v_i\in\{-1,1\}} \exp\left(\beta\left(u(v_i)+Jv_i\overline{m}_i^e\right)\right)}$

For $\beta \rightarrow \infty$, the random term becomes unimportant and it is a deterministic model. For $\beta \rightarrow 0$, the probability of choosing one action will converge to 0.5.

Since the error terms are independent across agents, the joint probability is given by:

$$\Pr(\omega) = \frac{\exp\left(\beta\left(\sum\limits_{i=1}^{I}(u(\omega_i)+J\omega_i\overline{m}_i^e)\right)\right)}{\sum\limits_{v_1\in\{-1,1\}} \cdots \sum\limits_{v_I\in\{-1,1\}} \exp\left(\beta\left(\sum\limits_{i=1}^{I} u(v_i)+Jv_i\overline{m}_i^e\right)\right)}$$

Example 1 *An example for $I = 2$:*

$\Pr(\omega_1,\omega_2) =$

$$\frac{\exp(\beta(u(\omega_1)+J\omega_1\overline{m}_1^e))}{\exp\left(\beta\left(u(-1)-J\overline{m}_1^e\right)\right)+\exp\left(\beta\left(u(1)+J\overline{m}_1^e\right)\right)} * \frac{\exp(\beta(u(\omega_2)+J\omega_2\overline{m}_2^e))}{\exp\left(\beta\left(u(-1)-J\overline{m}_2^e\right)\right)+\exp\left(\beta\left(u(1)+J\overline{m}_2^e\right)\right)}$$
$$= \frac{\exp\left(\beta\left(\sum\limits_{i=1}^{2}(u(\omega_2)+J\omega_2\overline{m}_2^e)\right)\right)}{\exp\left(\beta\left(u(-1)-J\overline{m}_1^e+u(-1)-J\overline{m}_2^e\right)\right)+\exp\left(\beta\left(u(-1)-J\overline{m}_1^e+u(1)+J\overline{m}_2^e\right)\right)+\exp\left(\beta\left(u(1)+J\overline{m}_1^e+u(-1)-J\overline{m}_2^e\right)\right)+\exp\left(\beta\left(u(1)+J\overline{m}_1^e+u\right.\right.}$$

$$\frac{\exp\left(\beta\left(\sum\limits_{i=1}^{2}(u(\omega_2)+J\omega_2\overline{m}_2^e)\right)\right)}{\sum\limits_{\nu_1\in\{-1,1\}}\sum\limits_{\nu_2\in\{-1,1\}} \exp\left(\beta\left(\sum\limits_{i=1}^{2}\left(u(\nu_i)+Jv_i\overline{m}_i^e\right)\right)\right)}$$

Since there are only two choices private utility can be written as follows:
$h + k = u(1)$
$-h + k = u(-1)$
$\Rightarrow h = 0.5\,(u(1) + u(-1))$

Using this normalization, one has the following formula for the expected value of the choice:

$$E(\omega_i) = 1 * \frac{\exp\left(\beta\left(h+J(I-1)^{-1}\sum\limits_{j\neq i}\overline{m}_{i,j}^e\right)\right)}{\exp\left(\beta\left(h+J(I-1)^{-1}\sum\limits_{j\neq i}\overline{m}_{i,j}^e\right)\right)+\exp\left(\beta\left(-h-J(I-1)^{-1}\sum\limits_{j\neq i}\overline{m}_{i,j}^e\right)\right)}$$

$$-1 * \frac{\exp\left(\beta\left(-h-J(I-1)^{-1}\sum_{j\neq i}\overline{m}^{e}_{i,j}\right)\right)}{\exp\left(\beta\left(h+J(I-1)^{-1}\sum_{j\neq i}\overline{m}^{e}_{i,j}\right)\right)+\exp\left(\beta\left(-h-J(I-1)^{-1}\sum_{j\neq i}\overline{m}^{e}_{i,j}\right)\right)}$$

Remember definition of tangens hyperbolicus:
$\tanh(x) = \frac{\exp(x)-\exp(-x)}{\exp(x)+\exp(-x)}$

The expected value can then be also written as:
$E(\omega_i) = \tanh\left(\beta h + \beta J (I-1)^{-1} \sum_{j\neq i} \overline{m}^{e}_{i,j}\right)$

Finally, the authors impose rational expectations and require that
$\overline{m}^{e}_{i,j} = E(\omega_j)\, \forall i, j.$

In the presence of multiple equilibria this leaves the question how the agents will coordinate their expectations on the 'right' equilibrium.

$E(\omega_i) = \tanh\left(\beta h + \beta J (I-1)^{-1} \sum_{j\neq i} E(\omega_j)\right)$

Notice that $E(\omega_i) \in [-1, 1]$. Since the tanh-function is a continuous mapping, Brouwer's fixed point theorem applies.

Proposition 2 *(Existence of self-consistent equilibrium for discrete choices with noncooperative decisionmaking) When agents choose actions noncooperatively given social utility specification (proportional spillovers case) and given self-consistent expectations, there exists at least one expected average choice level m^* such that*

$$m^* = \tanh(\beta h + \beta J m^*)$$

With strategic complementarities there is the possibility of mulitple equilibria

Proposition 3 *(Existence of multiple average choice levels in equilibrium).*

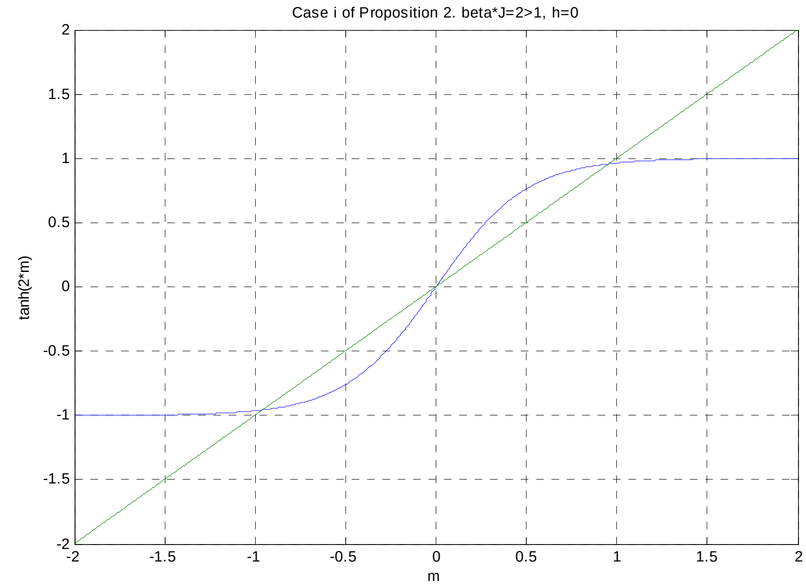
1. *(i) If $\beta J > 1$ and $h = 0$, there exist three roots of $m^* = \tanh(\beta h + \beta J m^*)$. One of these roots is positive, one root is zero, and one root is negative.*

   (ii) If $\beta J > 1$ and $h \neq 0$, there exists a threshold H, (which depends on βJ) such that

   (a) for $|\beta h| < H$, there exist three roots to $m^* = \tanh(\beta h + \beta J m^*)$, one of which has the same sign as h, and the others possessing opposite sign

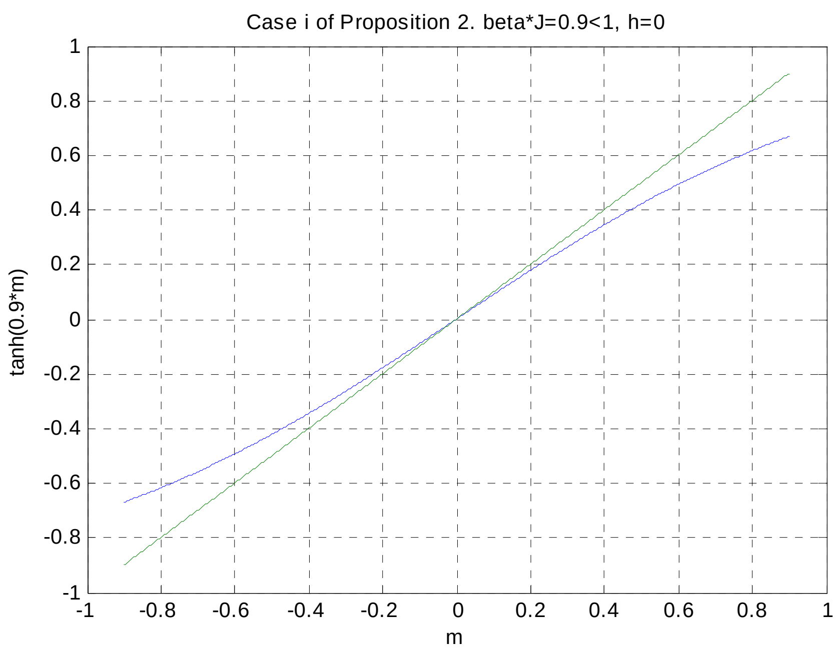
   (b) for $|\beta h| > H$, there exists a unique root to $m^* = \tanh(\beta h + \beta J m^*)$ with the same sign as h.

Intuition for these results: In the absence of a bias for one choice and strong enough social interactions one has three possible equilibria (case i)



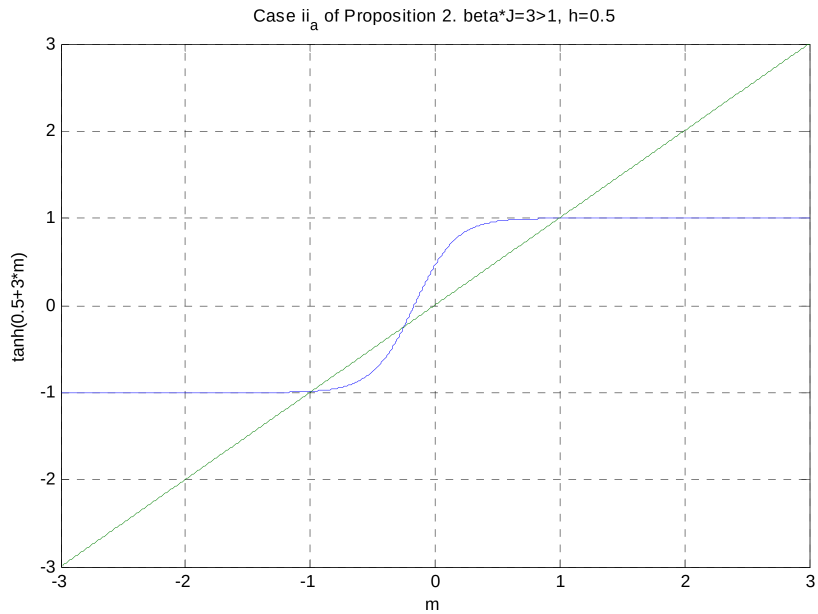
Case (i) with strong social interactions

If the social interactions are not strong enough $\beta J < 1$ one has the following



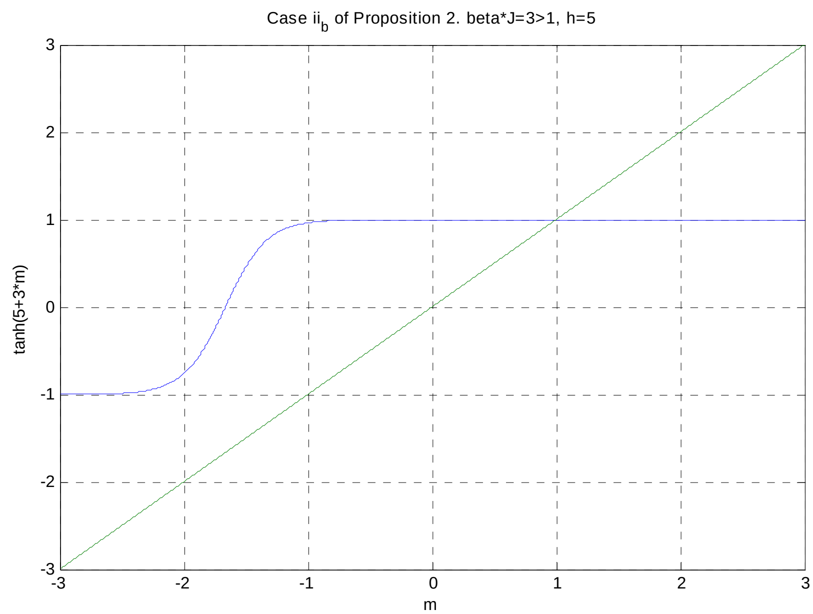
Case with weaker social interactions

Finally, if the private utility induces an a priori bias these graphs are not symmetric around zero anymore.



Case (iia): Private utility and relatively strong social interactions

With private utility the bias towards one action can be strong enough to overcome the social interactions effects:



Case (iib): Private utility and relative weak social interactions

Notice that in the latter case the social interactions effect is actually stronger than the social interactions in the case (i). For latter reference they also introduce even more notation:

m^*_-, m^*_m, m^*_+ refer to the different equilibria.

1.3.4 Stability: just think of the Solow growth model

They consider the dynamic stability of m^*_-, m^*_m, m^*_+ under the assumption that all variables are now subscribed by time and the expectations term obeys

$\overline{m}^e_{i,t} = m^*_{t-1}$

That is in this model, agents are myopic. One can demonstrate the stability using the graphs above and using the 45-degree line just like back in the days one showed that the steady state capital stock in the Solow-growth model is stable. This consideration also demonstrates why the unique equilibrium is always stable. If there are three equilibria, then only the two extremes are stable. In the remainder they only discuss the stable equilibria.

1.3.5 Pareto ranking of equilibria

Only ex-ante ranking is possible. On an ex-ante basis if private utility is positive then m^*_+ is the more desirable equilibrium and vice versa. If the private utility term is zero then both equilibria have the same welfare properties. One can immediately see here, that the noncooperative outcome could be improved upon by a social planner. Say the private utility from dropping out of high-school is actually negative but we are stuck in an equilibrium where a majority drops out, then we would be collectively better off by not dropping out and move to another equilibrium. However, it is individually rational to drop out. Classical problem: In the presence of externalities there may be ways to make everyone better off.

1.4 Social Planner

Average choice level is unique under the social planner.

With proportional spillovers:

> This means that even if the mean choice level in the noncooperative equilibrium has the same sign as the social planner equilibrium, the average choice in the noncooperative case will still be socially inefficient. (p. 246)

With conformity effects:
Pareto efficient equilibrium is sustained under decentralized decisionmaking.

1.5 Econometrics

Discuss identifiability of this model.

2 Discussion

1. Basically, still a static model. Why use self-consistent expectations under decentralized decisionmaking? How can agents coordinate their expectations.
2. Very specific functional forms: how robust are the specifications
3. Timing of the choice problem: simultaneous choices?