Notes on Duggan and Martinelli "A Bayesian Model of Voting in Juries"
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The paper considers situations where groups of agents sharing a common goal must vote on a binary decision on the basis of private information. Applications include corporate boards of directors, medical panel, and most notably, juries.

1 The Nature of Private Information

How should one model the private information of jury members? Previous studies, including Feddersen and Pesendorfer (1998), assume that agents receive draws from a discrete distribution typically indicating only an "innocent" or "guilty" signal. Duggan and Martinelli seek to replace this signal mechanism with one producing signals from a continuous distribution. There are two reasons:

1. Realism
2. Equilibrium characteristics: Using discrete signals, equilibria often require some jurors to employ mixed strategies. It may be desirable to create a model with pure strategy equilibria.

2 The Model

2.1 Primitives

- Jury size of $n \geq 2$ Let N denote the set of jurors.
- The defendant is either innocent, I, or guilty, G with probabilities $P(I)$ and $P(G)$
- Each juror receives a signal, s_i, which is drawn from either $F(\cdot|I)$ or $F(\cdot|G)$ depending on the innocence or guilt of the defendant.
- **Procedure**: Jurors receive their signals and then simultaneously submit their votes.
- **Decision Rule**: Let n_c represent the number of votes for conviction. If $n_c \geq k$, then the defendant is convicted, C. Otherwise, the defendant is acquitted, A.
- **Utility Functions**: $u(C|G) = u(A|I) = 0 \quad u(A|G), u(C|I) < 0$ This allows one to compute some useful quantities. $u(C|I)P(I)$ is the ex ante expected cost of conviction, and $u(A|G)P(G)$ is the ex ante expected cost of acquittal. Thus, $\rho = \frac{u(A|G)}{u(C|I)} \frac{P(G)}{PI}$ represents the relative ex ante cost of acquittal.
- **Strategies**: $\sigma_i : \Re \to [0,1]$, where $\sigma_i(s_i)$ is the probability that agent i will convict given their signal. This allows the calculation of the probability that juror i will convict conditional

only on guilt or innocence:

$$\begin{aligned}P^i_{\sigma_i}(C|I) &= \int \sigma_i(s)\mu_I(ds)\\ P^i_{\sigma_i}(C|G) &= \int \sigma_i(s)\mu_G(ds)\end{aligned}$$

where μ_I and μ_G are induced by $F(\cdot|I)$ and $F(\cdot|G)$

2.2 Payoffs

- Let σ denote the vector of strategies used by the n jurors. Given this strategy the probability of conviction given innocence is:

$$P_\sigma(C|I) = \sum_{\substack{M\subseteq N\\ |M|\geq k}} \prod_{j\in M}(\int \sigma_i(s)\mu_I(ds)) \prod_{j\notin M}(\int [1-\sigma_i(s)]\mu_I(ds)) \tag{1}$$

Similarly, the probability of acquittal given guilt is:

$$P_\sigma(A|G) = \sum_{\substack{M\subseteq N\\ |M|< k}} \prod_{j\in M}(\int \sigma_i(s)\mu_G(ds)) \prod_{j\notin M}(\int [1-\sigma_i(s)]\mu_G(ds)) \tag{2}$$

- Hence, the payoff to any juror from profile σ is given by:

$$\begin{aligned}PAYOFF &= P(I)[(0)(1-P_\sigma(C|I)) + u(C|I)P_\sigma(C|I)] + P(G)[(0)(1-P_\sigma(A|G)) + u(A|G)P_\sigma(A|G)]\\ &= P(I)u(C|I)P_\sigma(C|I)P(I) + u(A|G)P_\sigma(A|G)P(G)\end{aligned}$$

- When optimizing, each juror must take into account the probability that they themselves will be pivotal, or that they will cast the deciding vote, conditional on the strategic decisions of the other players (σ_{-i}). Depending on guilt or innocence these are given by:

$$\begin{aligned}P_{\sigma_{-i}}(piv|I) &= \sum_{\substack{M\subseteq N\\ |M|=k-1,\ i\notin M}} \prod_{j\in M}(\int \sigma_i(s)\mu_I(ds)) \prod_{j\notin M,\ i\neq j}(\int [1-\sigma_i(s)]\mu_I(ds))\\ P_{\sigma_{-i}}(piv|G) &= \sum_{\substack{M\subseteq N\\ |M|=k-1,\ i\notin M}} \prod_{j\in M}(\int \sigma_i(s)\mu_G(ds)) \prod_{j\notin M,\ i\neq j}(\int [1-\sigma_i(s)]\mu_G(ds))\end{aligned}$$

- Proposition (1): Given σ_{-i}, the ex ante payoff to juror i from σ_i is an affine transformation of:

$$(\int \sigma_i(s)\mu_I(ds))u(C|I)P_{\sigma_{-i}}(piv|I)P(I) - (\int \sigma_i(s)\mu_G(ds))u(A|G)P_{\sigma_{-i}}(piv|G)P(G) \tag{3}$$

2.3 Equilibria

- Strategy profile σ constitutes an equilibrium if for every juror i and for every alternate strategy for i, σ_i':

  $P(I)u(C|I)P_\sigma(C|I)P(I) + u(A|G)P_\sigma(A|G)P(G) \geq P(I)u(C|I)P_{\sigma_i',\sigma_{-i}}(C|I)P(I) + u(A|G)P_{\sigma_i',\sigma_{-i}}(A|G)P(G)$

- A responsive equilibrium is one in which no juror convicts or acquits irrespective of their signal: $0 < \int \sigma_i(s)\mu_I(ds) < 1$, and $0 < \int \sigma_i(s)\mu_G(ds) < 1$

- Duggan and Martinelli are interested in finding cutoff equilibria in which each juror plays a cutoff strategy such that for some $\overline{s_i} \in [-\infty, \infty]$: $\sigma_i = 1$ if $s > \overline{s_i}$, $\sigma_i = 0$ if $s < \overline{s_i}$

- Duggan and Martinelli make the following four assumptions which allow them to focus only on cutoff equilibria:

  1. (A1) The distribution functions are absolutely continuous with respect to Lebesgue measure and have piecewise continuous densities $f(\cdot, I)$ and $f(\cdot, G)$
  2. (A2) The densities have common support, $S = (\underline{S}, \overline{S})$, $\underline{S}, \overline{S} \in [-\infty, \infty]$
  3. (A3) The likelihood ratio $\frac{f(s|I)}{f(s|G)}$ is weakly decreasing in s. At times it will be useful to assume that the likelihood ratio is locally strictly decreasing at some point.
  4. (A4) $\lim_{s \downarrow \underline{S}} \frac{f(s|I)}{f(s|G)} > \rho > \lim_{s \uparrow \overline{S}} \frac{f(s|I)}{f(s|G)}$

- . Consider the following function: $J(\sigma_{-i}, s) = \frac{P_{\sigma_{-i}}(piv|I)}{P_{\sigma_{-i}}(piv|G)} \frac{f(s|I)}{f(s|G)} - \rho$. $J(\sigma_{-i}, s)$ is positive if the expected payoff to a vote for acquittal exceeds the expected payoff to a vote for conviction. The function is weakly decreasing in s.

- . Lemma 1. Given responsive strategies σ_{-i}, a strategy σ_i is a best response for i if and only if it satisfies the following a.e.:

  $$\begin{aligned} \sigma_i(s) &= 1 \ if \ J(\sigma_{-i}, s) < 0 \\ \sigma_i(s) &= 0 \ if \ J(\sigma_{-i}, s) > 0 \end{aligned}$$

  If the likelihood ratio is strictly locally decreasing at $\inf(s \in S | J(\sigma_{-i}, s) \leq 0)$, then σ_i is a best response for i if and only if it is equivalent a.e. to the cutoff strategy given by:

  $$\begin{aligned} \widetilde{\sigma}_i(s) &= 1 \ if \ J(\sigma_{-i}, s) \leq 0 \\ \widetilde{\sigma}_i(s) &= 0 \ if \ J(\sigma_{-i}, s) > 0 \end{aligned}$$

- Theorem 1: There exists a symmetric responsive cutoff equilibrium with cutoff given by $s^* = \inf(s \in S | J(\sigma_{-i}, s) \leq 0)$, which is unique in the class of symmetric responsive cutoff profiles. If the likelihood ratio is locally strictly decreasing at s^*, then this equilibrium is unique a.e. within the class of all symmetric responsive profiles.

- Note that s^* is weakly decreasing in ρ.

3 Evaluating the Unanimity Rule

- Define hazard rates for when the defendant is innocent and guilty:

$$H(s|I) = \frac{f(s|I)}{1 - F(s|I)}$$
$$H(s|G) = \frac{f(s|G)}{1 - F(s|G)}$$

- Let s_n and s_k denote the cutoff values for the decision rules requiring n and k votes to convict, respectively.
- Lemma 3: If $n \geq k' > k \geq 1$, then $s_{k'} \leq s_k$
- Corollary 1: If $\frac{H(s|I)}{H(s|G)}$ is weakly increasing on an open interval including $[s_n, s_k]$, then

$$\frac{P_n(C|I)}{P_n(C|G)} > \frac{P_k(C|I)}{P_k(C|G)} \tag{4}$$