Preferences with a social component

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March 3, 2005

Abstract

I present a model of the simultaneous selection of goods or actions which demonstrate increasing returns to scale. Prior models of such goods universally describe people as basing their decisions upon the actions chosen by previous actors; the outcome depends as much on the sequence of actors as on the prior information decision makers have. Actors in the model here act simultaneously, so they must decide what to do based only on information about the distribution of tastes in the society. The shape of this distribution (e.g., centered around zero, skewed upward, or fat-tailed) predicts the number of people who will act in systematic ways. The model of Brock and Durlauf [2001] is a special case of the model described here.

1 Introduction

This paper presents a reduced-form model which describes situations in which people gain more utility from a good or action if others consume the same good or act in the same manner. It includes situations which different classes of literature describe as conformity, network effects, herding, fashion, or contagion. Specific examples include the consumption of operating systems, movies, or music; or even the decision to revolt against the government.

Existing models of these situations are sequential models, where people observe others before deciding to act. This paper presents a simultaneous model, and therefore arrives at results which differ in many ways from the extant literature.

Some preliminary notation will facilitate discussion. Let t be a real number representing a person's taste for an action if the person were not concerned about others. A negative number would imply that one prefers not to act. Let k be the percentage of people who are acting. Then a consumer's utility from acting will be $V(t, k)$, a function which is monotonically increasing in both t and k.

*Thanks to Richard McKelvey and Matt Jackson for extensive commentary. Thanks also to Kim Border, Peter Bossaerts, Peyton Young, Carla Van Besalaere, Daniel Ray Clendenning, and Elizabeth Maggie Penn for critique and suggestions.

1.1 Motivation

All but one of the results of this paper make no other assumptions about the form of the value function $V(t,k)$. On the one hand, that means that this paper offers no explanations as to why people choose to behave like others. It is an enthralling question, but is taken as an assumption to the model here, not a result.

On the other hand, this paper is applicable in a wide variety of situations. The information cascade model says that others' behavior may reveal information which raises the expected utility from selecting the same action; therefore a person's expected value from acting is a monotonically increasing function of their own tastes and the percent of other people acting—thus fitting the form $V(t,k)$. Goods which have 'network externalities', such as computer operating systems, have two components to their utility: a private utility and a benefit from networking with others, which may be expressed as $V(t,k)$. Goods traditionally thought of as fashion goods, such as music or clothing, clearly have a private and a public component, and can be expressed using the same reduced form.

1.2 Comparison with prior scholarship

1.2.1 The restaurant problem

The main contribution of this paper is to present a simultaneous model of belief formation and action. The most common comparable model is that of sequential 'information cascades', as described by Banerjee [1992] or Bikhchandani et al. [1992].

Since people act sequentially, the nth decision maker has $n-1$ signals that he may look to, meaning that he does not need to have well-developed priors.

There is almost no place in the herding model for comparing different prior beliefs. The true value from the action is taken to be constant throughout the society, although prior beliefs about t may differ. In the model here, each person knows his or her own private value t_i with certainty, but this differs from person to person. Since tastes are heterogeneous, people need to have prior beliefs about the distribution of tastes within the society. Let $f(t)$ be the probability density function (PDF) of tastes among the population. The information cascade models have no place for $f(t)$. In the model here, different priors on the characteristics of $f(t)$ (symmetric, skewed, fat-tailed) will affect the final percentage of people who will act.

The herding model remains untested in the literature. Tests searching for herding effects in various situations abound (e.g., Weinberg et al. [2001] on employment, Evans et al. [1992] on racial attitudes, Campbell [1980] on high school dropout and pregnancy rates, Kennedy [2002] on prime-time television, Anderson and Holt [1997] in the lab, et cetera), but none of these tests are informed by the models from the herding literature which they cite. The problem is not with the empirical tests, it is with the theoretical model, which does not make testable predictions beyond the basic claim that people herd. The model of this paper makes specific, testable claims about herding based on publicly available information and prior beliefs.

1.2.2 Brock and Durlauf's model

Brock and Durlauf [2001] have a similar model, but assume that $f(t)$ is a logistic distribution and that the value function is linear ($V(t,k) = t + nk$). Besides reducing the generality of their work, it also makes it impossible to make predictions based on the shape of the distribution. The results here go into detail about outcomes given different assumptions about $f(t)$.

Finally, Brock and Durlauf assume a cutoff-type equilibrium, while Lemma 3 will prove that cutoff equilibria can be derived from the other assumptions of the model.

1.3 Highlights from the results

We are interested in what I call *conviviality effects*. Active conviviality is a situation where an actor would not act if basing his decision entirely upon his private preferences, but chooses to act given knowledge or expectations about the behavior of his cohorts. Inactive conviviality is the opposite situation: an actor privately prefers to act, but refrains given information about others' choices. To the extent that conviviality effects are prevalent, demand will differ from the familiar demand curve implied by purely private utility.

Given the assumptions here, only cutoff equilibria are possible. In other words, if we order people by their private taste for a good, then only those whose taste is above a certain cutoff will choose to act. In this case, only one of active or inactive conviviality are possible, but the question remains which. Depending on the prior information held by the actors, there may be multiple equilibria demonstrating either effect. In this case, factors outside the model will determine which outcome prevails.

If the distribution of tastes is sufficiently diffuse, then there will be a unique equilibrium.

Finally, if we have some prior distribution of beliefs, then this will induce some posterior distribution of equilibria. But the posterior distribution of equilibria will be more diffuse than that of the prior distribution of tastes. As the desire to conform becomes more strong, the distribution of equilibrium outcomes will be more and more fat-tailed, until moderate equilibria with few conviviality effects will become impossible and only extreme equilibria will occur.

1.4 Outline

Section 2 defines the model. Section 3 defines a Nash equilibrium in this context. Section 4 discusses equilibria given complete information.

2 The model

Actors are faced with a binary action. For example: stand in line at the popular restaurant or go straight in to the unpopular one, install either a DOS-based operating system or a UNIX-based system, revolt or do not revolt, buy a good or do not buy it. For the sake of consistent terminology, I will describe the choice as being between 'acting' and 'not acting'.

A countably infinite number of individuals will simultaneously choose to act or not act. The decision is based on two factors. The first factor is the *ex post* percentage[1] of others acting, $k \in [0, 1]$, which will be endogenously determined. The second factor is the net individual utility to consumer i, denoted by a real number t_i. The distribution of t within the society depends on a parameter m, which is discussed below. Given m, the distribution of tastes is described by a PDF $f(t|m)$ which has support $(-\infty, \infty)$ for any given m.

2.1 Time line

The sequence of events is as follows:

1. Nature draws a fixed value μ for m, where m is distributed with PDF $a(\cdot)$; after the draw, μ is common knowledge.

2. Individuals draw a private utility t_i.

3. Person i now has enough information to calculate the expected utility from acting and from not acting, and makes a decision accordingly.

4. All individuals act simultaneously.

5. Person i receives a payoff based on t_i and the actual proportion of people who acted.

2.2 Assumptions about the utility function

The total value from acting is a function $V(t_i, k) : \mathbb{R} \times [0, 1] \to \mathbb{R}$, where $t_i \in \mathbb{R}$ is a level of tastes drawn from the distribution discussed above, and $k \in [0, 1]$ is the percentage of other actors, *ex post*, after everyone has decided whether to act or not.

Assumption 1 *The function $V(t, k)$ is the same for all actors, is continuous at all but a finite number of discontinuities, and is strictly monotonically increasing in both t and k.*

The utility from not acting is defined to be $V(0, 1 - k)$.

[1] This is an approximation, since it is impossible to define a percentage of a countably infinite number of people. More precisely, we will find that there exists a consistent estimate of the likelihood that a randomly drawn individual will act, $k \in [0, 1]$. I will stick to the description 'percentage of actors', however, because it makes more intuitive sense, and is formally correct in the real world, where there are a large but finite number of actors.

Notice that when not acting, people care about the percentage of others who are not acting, just as they care about the percentage of others who act when taking the action themselves.

We could have described the situation with three variables: t being the private utility from acting, k being the percent of actors, and s the private utility from not acting. But s would be redundant:

Lemma 1 *For any function $U(\cdot,\cdot)$ which is monotonically increasing in both arguments, there exists a function $V(\cdot,\cdot)$ which is also monotonically increasing in both arguments, such that $U(t,k) > U(s,1-k)$ iff $V(t-s,k) > V(0,1-k)$.*

[Proofs not given in the text can be found in the appendix, Section A.]

Normalizing the private utility from not acting to zero is therefore a convenient and unrestrictive assumption.

Finally:

Assumption 2 *There is some sufficiently low value $t_{\min}$ such that $V(t_{\min},1) < V(0,0)$.*

This means that if if i draws $t_i < t_{\min}$ then t_i will not act, regardless of expectations about k. In other words, tastes do matter somewhat, so that even if i knows that everyone will act with certainty ($k = 1$), he still will not act if he finds doing so sufficiently distasteful. This assumes away some perverse equilibria.

Having described the value from acting and not acting, the game is simply a question of comparing the expected values of V: all consumers observe their private utility t_i, and simultaneously decide whether to consume or not, based on whether $EV(t_i,k) > EV(0,1-k)$ or vice versa.

3 Equilibria defined

This section defines pure strategy symmetric Nash equilibria in the context of the model, along with the closely related expected demand function.

In the equilibria below, a person will be able to decide whether to act or not based entirely upon his or her draw of t_i. Let those t which prompt the consumer to act be T^A, and those that cause the consumer to not act as T^N. I will assume that consumers who are indifferent will not act; let the ts which induce indifference be T^*, meaning that $T^* \subseteq T^N$.

The actors in the model are completely described by the parameter t, and the set of pure-strategy actions available to them are to act or not act. Therefore, a pure strategy symmetric Nash equilibrium (herein referred to as 'an equilibrium') is completely described by a set T^A. For linguistic simplicity, I will often say 'T^A is an equilibrium' to mean 'in equilibrium, only members of the set T^A act'.

A set T^A is an equilibrium iff any individual who draws $t_i \in T^A$ chooses to act, given the prior information held by the individual and given that all other players are playing the strategy 'act iff I draw tastes $t_i \in T^A$'; and no individual who draws $t_i \notin T^A$ acts, given the same information.

Conviviality This notation allows the following definitions: active conviviality is when someone whose draw of t_i is less than zero chooses to act. That is, $T^A \cap (-\infty, 0) \neq \{\emptyset\}$. Inactive conviviality is when someone whose draw of t_i is greater than zero chooses not to act. That is, $T^N \cap (0, \infty) \neq \{\emptyset\}$.

The gist of these definitions is that if utility were purely a function of tastes, then everyone with $t_i < 0$ would not act while everyone who draws $t_i > 0$ would. Situations of active or inactive conviviality are those where someone has changed their behavior based on the behavior of others.

3.1 Cutoff equilibria and expected demand curves

A desirable refinement would be to have a cutoff equilibrium:

Definition 3 *A cutoff equilibrium is where T^A is of the form (T^*, ∞) for some point T^*.*

Lemma 3 will prove that all equilibria are cutoff equilibria, given the informational assumptions of this paper.

Define $k^*(t_i) : \mathbb{R} \to [0, 1]$ as the percentage of people who would need to act to make one who drew t_i indifferent between action and inaction. Let K^* be the equilibrium percentage of actors. If t_i has density $f(t)$, then let $F(t)$ be the cumulative distribution function (CDF) of t. Also, note that if t_i were the cutoff in a cutoff equilibrium, and the t_is are distributed $\sim F(\cdot|m)$, then $1 - F(t_i|m)$ percent will act.

Corollary 2 *The cutoffs for cutoff-type pure strategy Nash equilibria are at the value(s) of t_i for which $k^*(t_i) = 1 - F(t_i)$. If $k^*(t_i) > 1 - F(t_i)$ for all t_i, then $K^* = 1$ is an equilibrium (where $T^A = \emptyset$, meaning no one acts).*

In other words, we need only find a person who would be indifferent if they happened to be the cutoff. Such a person may consistently be a cutoff in equilibrium, and a person may only be a cutoff in equilibrium if he is indifferent given that he is the cutoff.

4 Equilibria found

First, we are guaranteed a cutoff equilibrium.

Lemma 3 *Given that $m = \mu$ is known, all pure strategy Nash equilibria are cutoff equilibria.*

From here, complications ensue; some examples may be helpful.

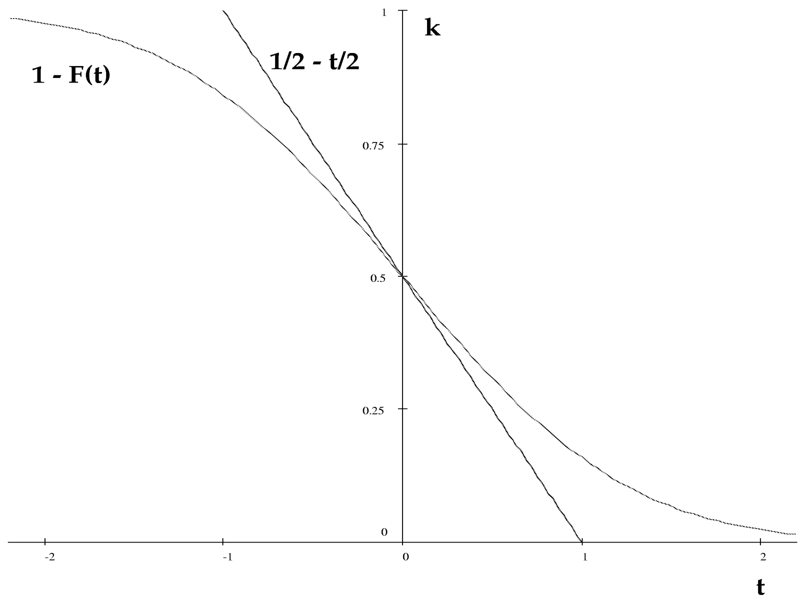

Figure 1: the 'normal' case

4.1 Example A: the normal case

Take $f(\cdot|m) = \mathcal{N}(m, 1)$, and take the value function to be linear and separable: $V(t, k) = t + nk$.

If $n = 1$, then we have the case in Figure 1: $k^*(t_i)$ is the straight line; $1 - F(t_i|0)$ is the curve.[2] The equation $k^*(t_i) = 1 - F(t_i|m)$ is satisfied at the crossing of these two curves, meaning that this is the location of any equilibria. Clearly, there is only one crossing point, and therefore a unique equilibrium.

Comparative statics are simple in this case: if m rises from zero to a positive value, the curve translates to the right, and therefore the cutoff falls. Since the cutoff falls, more people act. In other words, if it is revealed that more people like a good, then more people will consume.

4.2 Example B: multiple equilibria

The case $n = 2$, so $V(t, k) = t + 2k$, is much more interesting. Now consumers are twice as interested in the percentage of actors than in the good itself, and the result is as in Figure 2. There are three possible cutoffs, and the society

[2]What would it mean if $k^*(t_i) > 1$? Then a person would need more than 100% of his cohort to act before he is indifferent. In other words, he would never act. Since there can never be a negative percentage of actors, if $k^*(t_i) < 0$ then person i will always want to act.

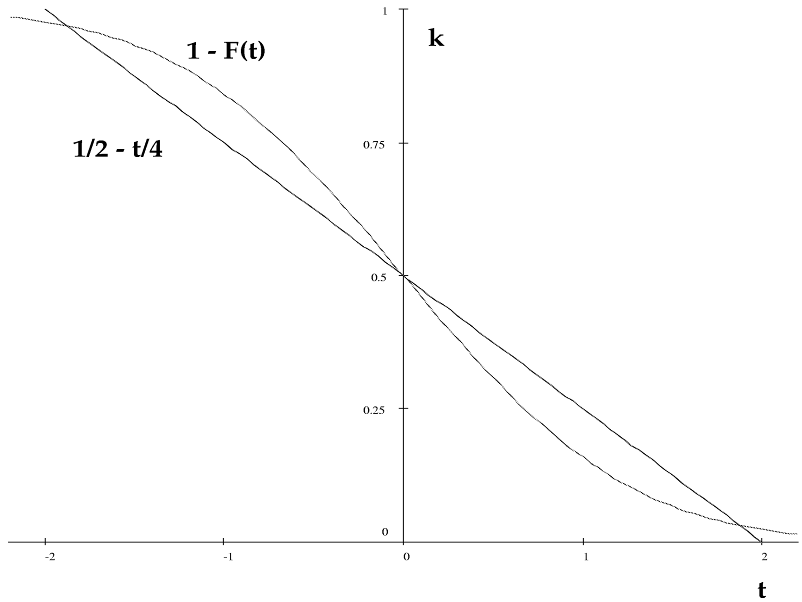

Figure 2: If people care enough about k, perverse equilibria appear.

agreeing on any one of them is a Nash equilibrium. One way to resolve this is to make a pessimism assumption, that given a few equilibria, everyone agrees on the one with the lowest number of people acting (and therefore the highest value of t).

The comparative statics for the pessimist (or the optimist) case is very simple: as the mean of the distribution rises, T^* falls and more people act. There is a jump in the value of the cutoff at a certain value (as the 'S' crosses the line).

Much more interesting is the 'moderation' case: if consumers agree upon the middle of the three equilibria, then as μ_A increases, T^* also increases, meaning that *fewer* people consume. People may coordinate on this middle equilibrium, but the equilibrium is unstable: if people think the cutoff is perturbed to the left of T^*, this would only lead to a greater shift to the left, eventually reaching the leftmost crossing point, and similarly with a shift to the right. Therefore the equilibrium point can be thought of as the dividing line between points which lead by *tatônnement* to a low T^* and points which lead to a high T^*; since this is increasing, the number of points which lead to high consumption rates increases as the mean taste for the action or good increases, consistent with intuition.

General results These examples show that if the distribution of tastes is diffuse enough or (equivalently) the desire for conformity is low enough, then there will be only one equilibrium. The following results explain sufficient conditions for one equilibrium.

Proposition 4 *The PDF $f(\cdot|m)$ may take any form, and the parameter m will be known to have the value μ with certainty. If for any equilibrium point $t = \tau$, $f(t|\mu) < -\frac{dk^*}{dt}(t)$ for all $t > \tau$, then there will be only one equilibrium for the given μ.*

Theorem 5 *The PDF $f(\cdot|m)$ may take any form, and the parameter m will be known to have the value μ with certainty. If $f(t) < -\frac{dk^*}{dt}(t')$ for all t, t', then there will be only one equilibrium for the given value of μ.*

The left-hand side of the inequality in this statement is the derivative of the CDF, while the right-hand side is the derivative of $k^*(t)$, so these statements hold when the CDF is expanding more slowly than $k^*(t)$ is contracting [Notice that dk^*/dt is always negative]. In words, this means that the value from conforming expands quickly relative to the most concentrated parts of the distribution. If the PDF has low peaks, or if $k^*(t)$ falls very quickly, then there will be only one equilibrium. This is sufficient for a unique equilibrium, but not necessary.

4.3 The distribution of cutoffs

Even though m was taken as fixed above, it may have been drawn from a distribution $a(m)$ before being fixed and made common knowledge. For example, the median of movie enjoyability may be observed to have distribution $a(m)$. When a new movie comes out, the median of its quality is drawn from $a(m)$, and reviewers reveal the value of m to the viewing public. The model is then as above: m is a fixed, known scalar, and Theorem 5 showed that if certain conditions hold, then $T^*(m)$ is a function producing only one equilibrium for each fixed value of m.

Let $T^*(\mu)$ be the function mapping values of μ to the equilibrium T^* that they induce.

Given the original *ex ante* distribution of $a(m)$, then, what is the *ex ante* distribution of T^*? The following theorem then says that under appropriate conditions, the distribution of T^* is a fat-tailed one relative to $a(m)$.

Theorem 6 *The ex ante distribution of the median m is $a(m)$, which can take any form. After drawing μ from $a(m)$, μ is known with certainty. Assume the PDF $f(x)$ is single-peaked, with a maximum value of $f(x) = \frac{1}{2n}$, and the value function is linear: $V(t, k) = nk + t$. Let $d(T^*)$ be the ex ante distribution of T^* (before μ is known).*

Given these assumptions, the ratio $a(m)/d(T^*(m))$ is single-peaked with a peak at the point where $\mu = T^*(\mu)$. That is, $d(T^*)$ is less concentrated toward the center than $a(m)$.

Given the assumptions about $f(\cdot)$ and $V(t, k)$ the conditions of Theorem 5 now apply, so there will be only one cutoff for any given value of m, and $T^*(m)$ is truly a function.

Returning to the example, if movie quality is normally distributed, then movie returns will be fat-tailed. De Vaney and Walls (Vaney and Walls [1996], p 1512) studied movie returns and found exactly this: "The end-of-run or total revenue distribution for motion pictures ... never quite reaches log normality; it has fatter tails than the log normal and mass points at the far right, where the superstars are located."

Continuing the example in the context of the theorem, $a(m)/d(T^*(m))$ is single-peaked, so its inverse $[d(T^*(m))/a(m)]$ can be described as 'single-troughed'. Also, Theorem 8 below will show that $T^*(0) = 0$, so this is where the nadir of the trough will be located. These facts imply that the distribution $d(T^*)$ must have a lower center and fatter tails than the Normal distribution. That is, actual movie quality which is distributed log-normally leads to log movie returns which have a leptokurtic distribution, as found by De Vaney and Walls.

An example For further intuition on the meaning of Theorem 6, consider the case where the distribution $a(m)$ is the Uniform$[-1, 1]$ distribution, and where $f(\cdot|m)$ is a Normal$(m, 1)$ distribution. Continue to assume that the value function is linear: $V(t, k) = nk + t$. Then Equation 8 of the proof (page 15) shows that the distribution of $d(T^*)$ is proportional to the ratio

$$\frac{\frac{1}{2n}}{\frac{1}{2n} - f(T^*(m)|m)}.$$

The premises of Theorem 5 assure us that the denominator is always positive, which would make this a continuous function of n for the range where these premises hold. When the distribution $f(t|m)$ is $\mathcal{N}(0, 1)$, the peak of the distribution is at zero, where $f(0|0) \approx .4$. If $\frac{1}{2n} >> .4$ [i.e., $n << 1.25$], this ratio as a function of m is approximately constant. If $\frac{1}{2n}$ is only a little larger than .4, the ratio will be larger at larger values of $|m|$ and dip to a minimum at $m = 0$. As n gets larger, the trough gets deeper, until $\frac{1}{2n} = .4$, at which point the function becomes discontinuous, multiple equilibria are possible, and only extreme values of T^* will be stable equilibria.

4.4 Giffen goods

Fashion goods are often used as an example of Giffen goods, so it is worth considering when Giffen goods would arise in the model here. It is easy to modify the model to include prices: take the parameter m to be the negation of the market price, so as the price rises, then the median of the consumers' value for purchasing the good falls. For example, if the good were free, then the taste for the action of purchasing the good may be distributed $\sim \mathcal{N}(0, 1)$, but if the good cost two dollars, then tastes for the action of purchasing the good would be distributed $\sim \mathcal{N}(-2, 1)$, with most people not wanting to consume the good, while those who strongly preferred the good when it was free still gaining positive utility from buying the good for \$2.

The question of drawing the demand curve them becomes: as the final price μ increases, does the cutoff T^* rise or fall?

The setup above gave conditions where $T^*(m)$ is a one-to-one function. Returning to the original interpretation where $f(t|m)$ is increasing in m,

Proposition 7 *Given the premises of Theorem 5, $T^*(\mu)$ is a strictly decreasing function.*

This reverses if m represents prices, so $f(t|m)$ is decreasing in m: as the fixed price μ rises, the cutoff T^* rises, meaning that the percentage of consumers falls. In other words, given the premises of Theorem 5, the demand curve always slopes down.

The premises of Theorem 5 are not trivial, and all of them may be plausibly dropped to describe a society where Giffen goods could exist. First, it is assumed that either consumers are not too concerned with the behavior of others, or that tastes are not too concentrated. Fashion goods are universally those where people are concerned with emulating the behavior of others. Also, if there are multiple equilibria, then a shift in prices can both cause a change in the location of the equilibria and a change in which equilibrium the consumers coordinate upon.

Second, it is assumed that the value of acting is monotonically increasing in the percentage of other actors. But consider fashion good models such as Frijters [1998], Pesendorfer [1995], or Bernheim [1994], based on goods as a signal of status. Each of these models feature a utility function which is increasing in the percentage of high-type consumers of a good, and is decreasing in the percentage of low-type consumers. Therefore the value of consuming may increase or decrease as a function of the total number of consumers. The moral is not that fashion goods can not be Giffen goods—there is evidence that such goods do exist—but that any model of fashion goods with Giffen demand curves can not be based on a homogenous society with full information and purely self-interested actors.

4.5 Comparing posteriors

This section gives one more way in which the location of the cutoffs are related to the distribution of tastes in the society.

Definition 4 *A distribution $f(\cdot|\mu)$ is* symmetric *iff $f(d|\mu) = f(-d|\mu)\ \forall d$.*

A distribution $f(\cdot|\mu)$ is upward-leaning *iff $f(d|\mu) > f(-d|\mu)$ for all $d > 0$.*

A distribution $f(\cdot|\mu)$ is downward-leaning *iff $f(d|\mu) < f(-d|\mu)$ for all $d > 0$.*

For example, $d \sim \mathcal{N}(2,1)$ is an upward-leaning distribution, even though it is symmetric around two.

Theorem 8 *Given that $m = \mu$ is known.*

(i: no conviviality) In the symmetric case, there is an equilibrium at $T^* = 0$, meaning that there is neither active nor inactive conviviality.

(ii: active conviviality) If the distribution is upward-leaning, then there is an equilibrium at some point $T^* < 0$.

(iii: inactive conviviality) If the distribution is downward-leaning, then there is an equilibrium at some point $T^* > 0$.

None of these cases preclude other equilibria existing elsewhere, as shown by example B above, which was symmetric but had equilibria greater than, less than, and equal to zero. However, in the case where there is only one equilibrium, this demonstrates that the shape of the distribution matters in determining the final level of consumption, where upward-leaning distributions lead to more consumption; and the news of some ms are 'better' in the sense of causing more people to act.

5 Conclusion

People gain utility from behaving like others in a multitude of situations. If all individuals act simultaneously, however, the problem of predicting how others will behave typically requires more information than any one person has.

In the case where tastes are distributed normally, adding the fact that people like to behave like others does not change behavior from the entirely private valuation case, where information is limited. Without additional information, individuals are forced to assume that the average is like them, and the no-change result follows. However, they may switch to a new equilibrium after they know how many others actually are consuming. The new equilibrium will be more likely to be extreme the more consumers care about how others act (or equivalently, if the distribution of tastes has a tall peak); if consumers care enough, then *only* extreme equilibria are possible.

A Appendix

Following are the proofs of the results presented in the text. Lemma 9 and Lemma 10 are also stated, for use in subsequent proofs.

Lemma 9 *Let T_i^* be the equilibrium cutoff given m_i. If $m_1 \neq m_2$, then $T_{m1}^* \neq T_{m2}^*$.*

Proof:

Assume $m_1 > m_2$, and that $T_{m1}^* = T_{m2}^*$. Since $T_{m_1}^*$ and T_{m2}^* are assumed to be the same, let T^* signify both. One who drew $t_i = T^*$ would be indifferent between action and inaction given either m_1 or m_2, meaning that

$$V(T^*, 1 - F(T^*|m_1)) = V(0, F(T^*|m_1)) \quad (1)$$

and

$$V(T^*, 1 - F(T^*|m_2)) = V(0, F(T^*|m_2)) \quad (2)$$

But $m_1 > m_2$ and the monotonicity of $V(t,k)$ immediately implies that $1 - F(T^*|m_1) > 1 - F(T^*|m_2)$, so

$$\begin{aligned} V(T^*, 1 - F(T^*|m_1)) &> V(T^*, 1 - F(T^*|m_2)) \\ &\text{and} \\ V(0, F(T^*|m_2)) &> V(0, F(T^*|m_1)) \end{aligned}$$

meaning that only one of Equations 1 or 2 can be true. $\diamondsuit$

Lemma 10 *Say two lines, defined by $y = ax + k_a$ and $y = bx + k_b$, intersect at the point (x_1, y_1). Then a one unit horizontal shift in the first line, to $y = a(x-1) + k_a$ leads to a horizontal shift in the point of intersection of $\frac{a}{b-a}$, so the x-coordinate of the new point of intersection is $x_1 + \frac{a}{b-a}$.*

Proof: We can solve the initial system of equations to find that the point of intersection for $y = ax + k_a$ and $y = bx + k_b$ is at the horizontal coordinate

$$x_1 \equiv \frac{k_b - k_a}{a - b},$$

while the intersection for the lines $y = a(x-1) + k_a$ and $y = bx + k_b$ is at the horizontal coordinate

$$\begin{aligned} x_2 &\equiv \frac{k_b - k_a + a}{a - b} \\ &= x_1 + \frac{a}{a - b}, \end{aligned} \tag{3}$$

which was to be shown. $\diamondsuit$

Lemma 1 *For any function $U(\cdot,\cdot)$ which is monotonically increasing in both arguments, there exists a function $V(\cdot,\cdot)$ which is also monotonically increasing in both arguments, such that $U(t,k) > U(s, 1-k)$ iff $V(t-s,k) > V(0,1-k)$.*

Proof: Let $k_1^*(t,s)$ be the value of k such that $U(t, k_1^*(t,s)) = U(s, 1 - k_1^*(t,s))$. Let $\phi(t,s) = 1 - 2k_1^*(t,s)$, and let $V(\tau,k) = \tau + k$. Substituting $\phi(t,s)$ for τ, we see that a decision maker using V is indifferent when $1 - 2k_1^*(t,s) + k = 1 - k$, or when $k = k_1^*(t,s)$. That is, a decision maker using V and the transformed taste parameter $\tau = \phi(t,s)$ is indifferent when he would have been indifferent using the original utility function.

The reader may verify that the decision maker would also prefer acting or not acting in the same manner using both functions.

Finally, notice that $\phi(t,s)$ is monotonically increasing in t and monotonically decreasing in s, and $V(\tau,k) = \tau + k$ is monotonically increasing in τ. $\diamondsuit$

Corollary 2 *The cutoffs for cutoff-type pure strategy Nash equilibria are at the value(s) of t_i for which $k^*(t_i) = 1 - F(t_i)$. If $k^*(t_i) > 1 - F(t_i)$ for all t_i, then $K^* = 1$ is an equilibrium (where $T^A = \emptyset$, meaning no one acts).*

Proof: It is by definition necessary that the person at the cutoff be indifferent between acting and not acting, so at the cutoff, $t^*(t_i) = t_i$. This is sufficient because everyone who draws a t_i such that $k^*(t_i) > K^*$ will want to act and everyone who draws t_i such that $k^*(t_i) < K^*$ won't, by FOSD and monotonicity.

There will always be some value of $K^* \in (-\infty, \infty]$ which is an equilibrium. There are only two possibilities: $k^*(t_i) = 1 - F(t_i)$ at some number of points, in which case these are all potential equilibria; or $k^*(t_i) > 1 - F(t_i)$ for all values of t_i, in which case setting $K^* > 1$ is a consistent cutoff (since it will inspire no one to act). The possibility that $k^*(t_i) < 1 - F(t_i)$ for all values of t_i, in which case everyone acts and $K^* < 0$, is precluded by Assumption 2. $\diamondsuit$

Lemma 3 *Given that $m = \mu$ is known, all pure strategy Nash equilibria are cutoff equilibria.*

Proof: The percentage of actors given a set of intervals T^A (not necessarily a cutoff) is know to be $k(T^A, \mu)$. Now say that a person with a draw of t_1 is indifferent between action and inaction, i.e.:

$$V(t_1, k(T^A, \mu)) = V(0, 1 - k(T^A, \mu)).$$

Then by the monotonicity of $V(\cdot, \cdot)$, it must be that for all people who draw $t_i < t_1$,

$$V(t_i, k(T^A, \mu)) < V(t_1, k(T^A, \mu)) = V(0, 1 - k(T^A, \mu)),$$

so they therefore prefer not acting over acting. For all people who draw $t_j > t_1$,

$$V(t_j, k(T^A, \mu)) > V(t_1, k(T^A, \mu)) = V(0, 1 - k(T^A, \mu)),$$

so they prefer to act. Therefore t_1 is a cutoff, and T^A must be (t_i, ∞).

If the cutoff is $t^*(t_i) = \infty$, this is defined to mean that for all t_i, $V(t_i, k(T^A, \mu)) < V(0, 1 - k(T^A, \mu))$. $\diamondsuit$

Proposition 4 *The PDF $f(\cdot|m)$ may take any form, and the parameter m will be known to have the value μ with certainty. If for any equilibrium point $t = \tau$, $f(t|\mu) < -\frac{dk^*}{dt}(t)$ for all $t > \tau$, then there will be only one equilibrium for the given μ.*

Proof: Consider a point $\tau' > \tau$. Given the conditions here, we are guaranteed that

$$-\int_{\tau}^{\tau'} f(t|\mu)dt > \int_{\tau}^{\tau'} \frac{dk^*}{dt}(t)dt. \tag{4}$$

At τ, $1 - F(\tau|\mu) = k^*(\tau)$, or

$$1 - \int_{-\infty}^{\tau} f(t|\mu)dt = \int_{-\infty}^{\tau} \frac{dk^*}{dt}(t)dt. \tag{5}$$

The sum of Inequality 4 and Equation 5 gives:

$$1 - \left[\int_{-\infty}^{\tau} f(t|\mu)dt + \int_{\tau}^{\tau'} f(t|\mu)dt\right] > \int_{-\infty}^{\tau} \frac{dk^*}{dt}(t)dt + \int_{\tau}^{\tau'} \frac{dk^*}{dt}(t)dt, \tag{6}$$

or more succinctly, $1-F(\tau'|\mu) > k^*(\tau')$. Therefore, given the assumptions here, no point $\tau' > \tau$ can not be an equilibrium.
$\diamondsuit$

Theorem 5 *The PDF $f(\cdot|m)$ may take any form, and the parameter m will be known to have the value μ with certainty. If $f(t) < -\frac{dk^*}{dt}(t')$ for all t, t', then there will be only one equilibrium for the given value of μ.*

Proof: Given the condition here, the conditions of Proposition 4 will hold for any draw of μ. $\diamondsuit$

Theorem 6 *The ex ante distribution of the median m is $a(m)$, which can take any form. After drawing μ from $a(m)$, μ is known with certainty. Assume the PDF $f(x)$ is single-peaked, with a maximum value of $f(x) = \frac{1}{2n}$, and the value function is linear: $V(t,k) = nk + t$. Let $d(T^*)$ be the ex ante distribution of T^* (before μ is known).*

Given these assumptions, the ratio $a(m)/d(T^*(m))$ is single-peaked with a peak at the point where $\mu = T^*(\mu)$. That is, $d(T^*)$ is less concentrated toward the center than $a(m)$.

Proof: The transformation from the PDF $a(m)$ to the PDF $d(\cdot)$ is a simple coordinate transformation:

$$\begin{aligned} d(T) &= a(T^{*-1}(T))\frac{dT^{*-1}}{dm}(T) \\ &\text{or} \\ \frac{a(T^{*-1}(T))}{d(T)} &= \frac{dT^*}{dm}(m) \end{aligned} \tag{7}$$

A change in m is best envisioned using Figure 2 (page 8): a one-unit change in m translates the curve one unit to the left; $\frac{dT^*}{dm}$ is then how far to the left the intercept between the line and the curve moves. The slope of one minus the CDF is $-f(T^*(m)|m)$, and the slope of the function $k^*(m)$ is $-\frac{1}{2n}$. Plugging these slopes into Equation 3 from Lemma 10 (page 13), we get:

$$\frac{dT^*}{dm}(m) = \frac{\frac{1}{2n}}{\frac{1}{2n} - f(T^*(m)|m)}. \tag{8}$$

The premise of Theorem 5 guarantees that the denominator is always positive, meaning that a positive shift in m always leads to a finite, positive shift in T^*.

Equation 7 says that Equation 8 gives us the ratio $\frac{a(m)}{d(T^*(m))}$ which the Theorem describes. Since $f(\cdot)$ is assumed to be a single-peaked distribution, $f(T^*(m)|m)$ is largest where $T^*(m) = m$, and is monotonically decreasing as m diverges from that value in either direction; the left-hand side of Equation 8—and therefore the left-hand side of Equation 7—is also largest where $T^*(m) = m$ and is monotonically decreasing as m diverges from that point. $\diamondsuit$

Proposition 7 *Given the premises of Theorem 5, $T^*(\mu)$ is a strictly decreasing function.*

Proof: Let μ be the value of m which induces an equilibrium at τ, and let τ' be any value of t greater than τ. Recall Equation 6 from the proof of Theorem 5:

$$1 - \int_{-\infty}^{\tau'} f(t|\mu)dt > \int_{-\infty}^{\tau'} \frac{dk^*(t)}{dt}dt, \tag{9}$$

where μ is the median which leads to an equilibrium at $T^* = \tau$ and $\tau' > \tau$. Notice that the right-hand side is not a function of μ, while the left-hand side is a decreasing function of μ. Therefore, if the two sides of Inequality 9 were to be equal, they would be for some value $\mu' > \mu$, meaning that the equilibrium value $\tau' > \tau$ can only occur given a median $\mu' > \mu$. In other words, the mapping from m to T^* is an increasing function; along with the fact from Lemma 9 (page 12) that $T^*(m)$ is one-to-one, this means that its inverse $T^*(m)$ is itself an increasing function. ♢

Theorem 8 *Given that $m = \mu$ is known.*

(i: no conviviality) In the symmetric case, there is an equilibrium at $T^* = 0$, meaning that there is neither active nor inactive conviviality.

(ii: active conviviality) If the distribution is upward-leaning, then there is an equilibrium at some point $T^* < 0$.

(iii: inactive conviviality) If the distribution is downward-leaning, then there is an equilibrium at some point $T^* > 0$.

Proof: Recall that $t^*(0)$ is the point at which one who drew $t_i = 0$ would be indifferent if $t^*(t_0)$ were the cutoff. It is the point where

$$V(0, \int_{t^*(0)}^{\infty} f(x|m)dx) = V(0, 1 - \int_{t^*(0)}^{\infty} f(x|m)dx)$$

or

$$\int_{t^*(0)}^{\infty} f(x|m)dx = \int_{-\infty}^{t^*(0)} f(x|m)dx$$

In the symmetric case, where $f(x|m) = f(-x|m)$, this is clearly true at $t^*(0) = 0$, and therefore $T^* = 0$ does indeed describe a Bayesian Nash equilibrium.

In the case where $f(x|m) > f(-x|m)$, then the point which solves the above equation must satisfy $t^*(0) > 0$. By Assumption 2, there is some value of t, t_∞, such that the associated cutoff is $t^*(t_\infty) = -\infty$. Since $t^*(t_i)$ is a continuous, monotonic function of t_i, we know that $t_\infty < 0$, and that there is some point $t_e \in (t_\infty, 0)$ such that $t_e = t^*(t_e)$.

In the case where $f(x|m) < f(-x|m)$, then $t^*(0) < 0$. By continuity, there is some range such that $t^*(t_i) < t_i$ for all $t_i \in (0, t_e)$. If this range has an upper bound, then at that bound, $t_e = t^*(t_e)$; if it has no upper bound, then $T^* = \infty$ is an equilibrium. ♢

References

George A Akerlof and Rachel E Kranton. Economics and identity. Technical report, 1999.

Lisa R Anderson and Charles A Holt. Information cascades in the laboratory. *The American Economic Review*, 87(5):847–862, December 1997.

Abhijit V Banerjee. A simple model of herd behavior. *The Quarterly Journal of Economics*, 107(3):797–817, Aug 1992.

B Douglas Bernheim. A theory of conformity. *Journal of Political Economy*, 102(5):841–877, 1994.

Sushil Bikhchandani, David Hirshleifer, and Ivo Welch. A theory of fads, fashion, custom, and cultural change as informational cascades. *Journal of Political Economy*, 100(51):992–1026, 1992.

William A Brock and Steven N Durlauf. Discrete choice with social interactions. *Review of Economic Studies*, 68:235–260, 2001.

Bruce A Campbell. A theoretical approach to peer influence in adolescent socialization. *American Journal of Political Science*, 24(2):324–344, May 1980.

Michael Suk-Young Chwe. *Rational Ritual: Culture, Coordination, and Common Knowledge*. Princeton University Press, 2001.

William N Evans, Wallace E Oates, and Robert M Schwab. Measuring peer group effects: A study of teenage behavior. *Journal of Political Economy*, 100(5):966–991, October 1992.

Paul Frijters. A model of fashions and status. *Economic Modelling*, 15: 501–517, 1998.

Robert E Kennedy. Strategy fads and competitive convergence: An empirical test for herd behavior in prime-time television programming. *The Journal of Industrial Economics*, 50(1):57–84, March 2002.

Wolfgang Pesendorfer. Design innovation and fashion cycles. *The American Economic Review*, 85(4):771–792, September 1995.

Arthur De Vaney and W David Walls. Bose-Einstein dynamics and adaptive contracting in the motion picture industry. *The Economics Journal*, 47: 1493–1514, November 1996.

Bruce A Weinberg, Patricia B Reagan, and Jeffrey J Yankow. Do neighborhoods affect hours worked: Evidence from longitudinal data. Working paper, Ohio State University and Furman University, November 2001.